

The Crystal Structures of $\text{Na}_2\text{ZnSiO}_4$, $\text{Na}_2\text{ZnGeO}_4$, and $\text{Na}_2\text{MgGeO}_4$

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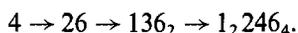
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The isotypic compounds $\text{Na}_2\text{ZnSiO}_4$, $\text{Na}_2\text{ZnGeO}_4$ and $\text{Na}_2\text{MgGeO}_4$ derive from orthorhombic $\beta\text{-NaFeO}_2$ ($\text{Pbn}2_1\text{-C}_{2v}^9$) by cross substitution of the type $\text{M}^{\text{II}} + \text{M}^{\text{IV}} = 2 \text{M}^{\text{III}}$ and their diffraction lines can be indexed with very similar orthorhombic unit cells. However the ordering of the $\text{M}^{\text{II}} = \text{Zn, Mg}$ and $\text{M}^{\text{IV}} = \text{Si, Ge}$ atoms on the $\text{M}^{\text{III}} = \text{Fe}$ sites lowers the symmetry to Pn or in the standard notation to Pc . The structure type of $\text{Na}_2\text{ZnSiO}_4$ is the first example of an ordered quaternary tetrahedral structure related to hexagonal diamond.

Introduction

It has been demonstrated, recently in great detail (Parthé, 1964), that normal tetrahedral structures can form if the average number of valence electrons per atom of the compound is equal to four. We understand under the term of a normal tetrahedral structure a structure where every atom in the structure has four nearest neighbor atoms located approximately at the corners of a surrounding tetrahedron. Using a simple procedure called cross-substitution it is possible, starting from diamond with the simplest tetrahedral structure, to derive theoretically all the possible binary, ternary, quaternary, and more complicated normal tetrahedral structure compositions. The cross-substitution consists of a replacement of an atom pair in the simple structure by two different elements having more and less electrons while keeping the electron to atom ratio constant. If we use the valence electron numbers of the elements to denote the elements we can demonstrate the principle of a cross-substitution by a series of compositions as for example;



We know two ideal tetrahedral structures for elements, one is the normal, cubic diamond structure, the other being the hexagonal diamond structure (Bundy and Kasper, 1967). The crystal structure of compounds where the compositions can be derived by cross-substitution should be essentially cubic or

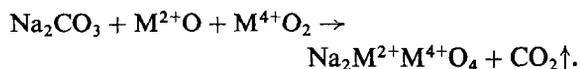
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hexagonal diamond structures with ordered atom arrangements. Examples are well known for structure types geometrically derived from normal diamond. They are the zinc blende structure type for 26 compounds, the chalcopyrite structure type for 136₂ compounds and the stannite structure type for 1₂246₄ compounds. Not so much is known for structure types derived from hexagonal diamond. The zincite (or wurtzite) structure type occurs with 26 compounds. Bertaut *et al.* (Bertaut and Blum, 1954; Bertaut, Delapalme, and Bassi, 1964) worked out the $\beta\text{-NaFeO}_2$ structure type for 136₂ compounds, but no ordered quaternary structure type related to hexagonal diamond has ever been found. Such a structure type could possibly occur with $\text{Na}_2\text{ZnSiO}_4$, $\text{Na}_2\text{ZnGeO}_4$ and $\text{Na}_2\text{MgGeO}_4$ and it was for this reason why the structural study of these compounds was undertaken.

Sample Preparation

The samples were prepared according to the reaction:



Mixtures of the oxides with soda were heated on air in porcellan boats. Heating time was 1 day at 500C plus 1 day at 900C plus 4 days at 1050C for $\text{Na}_2\text{ZnSiO}_4$, 3 days at 1100C for $\text{Na}_2\text{MgGeO}_4$ and 2 days at 700C plus 3 days at 1050C for $\text{Na}_2\text{ZnGeO}_4$. The reaction was complete and the final product contained no trace of other phases.

Structure Determination

The powder diffraction patterns of $\text{Na}_2\text{ZnSiO}_4$, $\text{Na}_2\text{ZnGeO}_4$, and $\text{Na}_2\text{MgGeO}_4$ show immediately

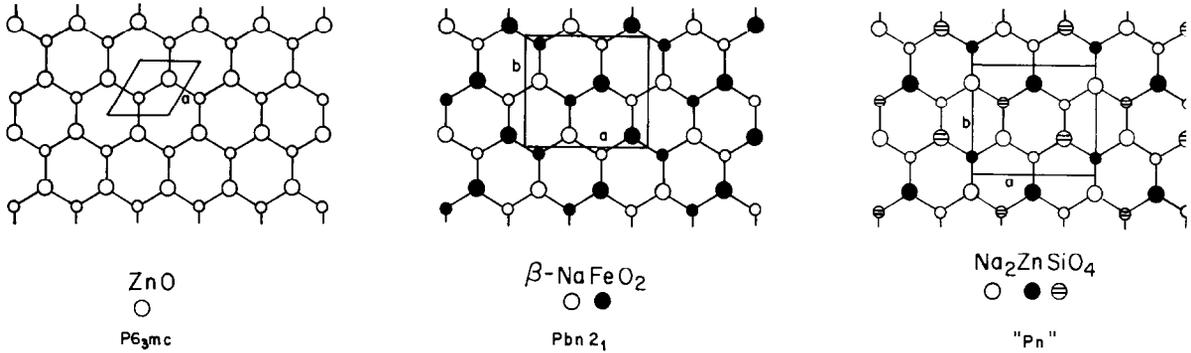


FIG. 1. Projections along the c axis for ZnO and the derivative structures of $\beta\text{-NaFeO}_2$ and $\text{Na}_2\text{ZnSiO}_4$. For clarity the oxygen atoms are not shown.

that all three compounds are isotypic and also very similar to $\beta\text{-NaFeO}_2$. Based on this similarity it was possible to index all lines of the new patterns with similar orthorhombic unit cells having the following parameters:

$$\begin{aligned} \text{Na}_2\text{ZnSiO}_4: & \quad a = 7.02 \text{ \AA} \quad b = 5.44 \text{ \AA} \quad c = 5.24 \text{ \AA} \\ \text{Na}_2\text{ZnGeO}_4: & \quad a = 7.17 \text{ \AA} \quad b = 5.56 \text{ \AA} \quad c = 5.32 \text{ \AA} \\ \text{Na}_2\text{MgGeO}_4: & \quad a = 7.15 \text{ \AA} \quad b = 5.60 \text{ \AA} \quad c = 5.35 \text{ \AA} \end{aligned}$$

In comparison the data for $\beta\text{-NaFeO}_2$ with setting $Pbn2_1(C_{2v}^9)$ are:

$$\beta\text{-NaFeO}_2: \quad a = 7.136 \text{ \AA} \quad b = 5.672 \text{ \AA} \quad c = 5.377 \text{ \AA}$$

While in $\beta\text{-NaFeO}_2$ all reflections $h0l$ with $h+l=2n+1$ and $0kl$ with $k=2n+1$ are systematically absent, in the patterns of the quaternary compounds weak $0kl$ reflections with $k=2n+1$ were observed, the only systematic extinctions being then $h0l$ with

TABLE I

UNIT CELL VECTORS AND IDEALIZED ATOM POSITIONS FOR THE TETRAHEDRAL STRUCTURE MODEL FOR $\text{Na}_2\text{ZnSiO}_4$

Pseudo orthorhombic cell "Pn"	First monoclinic cell Pc	Second monoclinic cell Pc
Unit cell vectors		
$\mathbf{a}_0 (\sim 2a_{\text{wurtzite}})$	$\mathbf{a}_{M_1} = \mathbf{a}_0$	$\mathbf{a}_{M_2} = \mathbf{a}_0 - \mathbf{c}_0 \left(\sim \frac{\sqrt{20}}{\sqrt{3}} a_{\text{wurtzite}} \right)$
$\mathbf{b}_0 (\sim \sqrt{3} a_{\text{wurtzite}})$	$\mathbf{b}_{M_1} = \mathbf{b}_0$	$\mathbf{b}_{M_2} = \mathbf{b}_0$
$\mathbf{c}_0 (\sim c_{\text{wurtzite}})$	$\mathbf{c}_{M_1} = -\mathbf{a}_0 + \mathbf{c}_0 \left(\sim \frac{\sqrt{20}}{\sqrt{3}} a_{\text{wurtzite}} \right)$	$\mathbf{c}_{M_2} = \mathbf{c}_0$
	$\text{tg} \beta = -\frac{a_0}{c_0}$ ($\beta \sim 140.8^\circ$)	$\text{tg} \beta = -\frac{c_0}{a_0}$ ($\beta \sim 129.2^\circ$)
Idealized parameters		
$xyz, x + \frac{1}{2} \bar{y}z + \frac{1}{2}$	$xyz, x\bar{y}z + \frac{1}{2}$	$xyz, x\bar{y}z + \frac{1}{2}$
2 Na with $x = \frac{1}{4}, y = \frac{3}{4}, z = 0$	2 Na with $x = \frac{1}{4}, y = \frac{3}{4}, z = 0$	2 Na with $x = \frac{1}{4}, y = \frac{3}{4}, z = \frac{1}{2}$
2 Na with $x = \frac{1}{2}, y = \frac{1}{6}, z = 0$	2 Na with $x = \frac{1}{2}, y = \frac{1}{6}, z = 0$	2 Na with $x = \frac{1}{2}, y = \frac{1}{6}, z = \frac{1}{2}$
2 Zn with $x = 0, y = \frac{1}{6}, z = 0$	2 Zn with $x = 0, y = \frac{1}{6}, z = 0$	2 Zn with $x = 0, y = \frac{1}{6}, z = 0$
2 Si with $x = \frac{1}{4}, y = \frac{3}{4}, z = 0$	2 Si with $x = \frac{1}{4}, y = \frac{3}{4}, z = 0$	2 Si with $x = \frac{1}{4}, y = \frac{3}{4}, z = \frac{1}{2}$
2 O with $x = \frac{1}{4}, y = \frac{3}{4}, z = \frac{3}{8}$	2 O with $x = \frac{1}{8}, y = \frac{3}{4}, z = \frac{3}{8}$	2 O with $x = \frac{1}{4}, y = \frac{3}{4}, z = \frac{3}{8}$
2 O with $x = \frac{1}{2}, y = \frac{1}{6}, z = \frac{3}{8}$	2 O with $x = \frac{1}{2}, y = \frac{1}{6}, z = \frac{3}{8}$	2 O with $x = \frac{1}{2}, y = \frac{1}{6}, z = \frac{3}{8}$
2 O with $x = 0, y = \frac{1}{6}, z = \frac{3}{8}$	2 O with $x = \frac{3}{8}, y = \frac{1}{6}, z = \frac{3}{8}$	2 O with $x = 0, y = \frac{1}{6}, z = \frac{3}{8}$
2 O with $x = \frac{1}{4}, y = \frac{3}{4}, z = \frac{3}{8}$	2 O with $x = \frac{5}{8}, y = \frac{3}{4}, z = \frac{3}{8}$	2 O with $x = \frac{1}{4}, y = \frac{3}{4}, z = \frac{3}{8}$

$h + l = 2n + 1$. The diffraction symmetry contained in the quaternary structures is thus only a n -glide perpendicular to the b direction, which, provided that the structure has orthorhombic symmetry, would indicate space group $Pmn2_1(C_{2v}^7)$ or $Pmnm(D_{2h}^{13})$ which is centro-symmetric and ruled out by a positive piezo electric test.

The similarity of the diffraction patterns to that of β -NaFeO₂ let us assume that the structure of the quaternary compounds is the same as that of β -NaFeO₂, except that now an Fe atom pair is replaced by one element of Group 2, like Zn or Mg, and one element of Group 4, like Si or Ge. In Fig. 1 are shown projections along the c axis for the ZnO structure, for the β -NaFeO₂ structure and for a model for the structure of Na₂ZnSiO₄, derived according to the above discussed scheme. For clarity the O atoms which are everywhere placed on top of the metal atoms have been omitted from the drawing. The model structure for Na₂ZnSiO₄ contains the n -glide perpendicular to the b -direction, however the symmetry is not anymore orthorhombic, but only pseudo-orthorhombic. If we insist on using the same orthogonal cell the apparent space group is Pn, which is a space group not listed as such in the International Tables. Agreement with the latter is obtained if a new set of monoclinic axes is chosen and the space group of Na₂ZnSiO₄ can then be written as Pc. There are two different monoclinic unit cells which could be chosen with equal justification. In Table I are listed the unit cell vector equations and the idealized point positions for the Na₂ZnSiO₄ model structure for the pseudo-orthorhombic and the two monoclinic unit cells.

A comparison of the powder diffraction intensities

with the intensities calculated for the structure model showed that the latter was essentially correct but needed refinement. To obtain accurate powder intensity values a diffractometer chart was recorded on drawing paper. Each diffraction peak was then cut out and the paper weighted on a microbalance. The intensity values corrected for Lorentz and polarization factor were then used in a special refinement program written by Dr. Bassi. The particular feature of the program is the possibility to use also intensity packets of coinciding diffraction lines, a feature which was of great importance here.

The final parameters for Na₂ZnSiO₄ with a reliability factor of 15% are shown in the left hand part of Table II. The agreement between calculated and observed d values and $p \cdot F^2$ values can be studied in Table III.

Comparison between the Different Structure Proposals

At this time in our research work we received notice of a Russian single crystal study on Na₂ZnSiO₄ by Ilyukhin, Nikitin, and Belov (1967). Their unit cell dimensions for Na₂ZnSiO₄ with "Pn" setting are:

$$a = 7.02 \text{ \AA} \quad b = 5.48 \text{ \AA} \quad c = 5.33 \text{ \AA} \quad \text{and} \quad \beta = 90^\circ 20'$$

in essential agreement with our data. Their atomic point positions, also with a reliability factor of 15%, are given on the right hand side of Table II. There is in general an agreement between our and the Russian results except for the z parameter of O_I and the y parameter of O_{IV}. These changes are, however, big enough to conceal the geometrical relationship of the quaternary structure to the hexagonal diamond

TABLE II

ATOMIC PARAMETERS FOR Na₂ZnSiO₄ (Setting "Pn")

This work				Ilyukhin, Nikitin and Belov (1967)		
			$xyz, x + \frac{1}{2}yz + \frac{1}{2}$			
2 Na _I	0.74 ₆	0.69 ₅	0.99 ₇	0.743	0.686	0.978
2 Na _{II}	0.50 ₅	0.17 ₇	0.99 ₇	0.504	0.185	0.980
2 Zn	0.00 ₃	0.19 ₂	0.99 ₉	0.000	0.168	0.000
2 Si	0.24 ₅	0.68 ₈	0.00 ₀	0.249	0.686	0.006
2 O _I	0.72 ₄	0.60 ₁	0.43 ₅	0.723	0.604	0.547
2 O _I	0.55 ₆	0.15 ₄	0.43 ₇	0.564	0.188	6.391
2 O _{III}	0.93 ₅	0.22 ₁	0.35 ₉	0.944	0.251	0.357
2 O _{IV}	0.28 ₆	0.68 ₉	0.30 ₂	0.283	0.800	0.281

TABLE III
 CALCULATED AND OBSERVED d -VALUES AND $p \cdot F^2$ VALUES FOR $\text{Na}_2\text{ZnSiO}_4$
 (CuK_α RADIATION)

hkl^a	hkl^b	d_{calc}	d_{obs}	$p \cdot F_{\text{calc}}^2 \cdot 10^{-2}$		$p \cdot F_{\text{obs}}^2 \cdot 10^{-2}$
010	010	5.44	5.45		0.9	1.0
11 $\bar{1}$	110, 1 $\bar{1}$ 0	4.30	4.30		70.6	
100	101	4.20	4.22	40.3	84.3	154.9
10 $\bar{2}$	10 $\bar{1}$			44.0		
011	011, 01 $\bar{1}$	3.774	3.78		20.5	16.1
20 $\bar{2}$	200	3.510	3.53		14.8	14.0
110	111, 1 $\bar{1}$ 1	3.324	3.33	11.1	18.2	13.7
11 $\bar{2}$	11 $\bar{1}$, 1 $\bar{1}$ 1			7.1		
21 $\bar{2}$	210, 2 $\bar{1}$ 0	2.949	2.957		143.4	150.6
020	020	2.720	2.712		180.4	175.6
002	002	2.620	2.634		254.1	262.3
21 $\bar{1}$	211, 2 $\bar{1}$ 1	2.570	2.575	232.7	457.5	462.5
21 $\bar{3}$	21 $\bar{1}$, 2 $\bar{1}$ 1			224.8		
12 $\bar{1}$	120, 1 $\bar{2}$ 0	2.536	2.533		5.0	524.8
021	021, 02 $\bar{1}$	2.414	2.412		90.8	72.8
012	012, 01 $\bar{2}$	2.361	2.360		6.8	3.4
120	121, 1 $\bar{2}$ 1	2.283	2.279	13.8	46.5	33.7
12 $\bar{2}$	12 $\bar{1}$, 1 $\bar{2}$ 1			32.7		
111	112, 1 $\bar{1}$ 2	2.237	2.239	7.7	47.6	38.5
11 $\bar{3}$	11 $\bar{2}$, 1 $\bar{1}$ 2			39.9		
22 $\bar{2}$	220, 2 $\bar{2}$ 0	2.150	2.149		26.2	89.8
31 $\bar{3}$	310, 3 $\bar{1}$ 0	2.149		22.7		
30 $\bar{2}$	301	2.136		38.4	40.9	75.6
304	30 $\bar{1}$			2.5		
200	202	2.100	2.104	22.8	53.8	36.7
204	20 $\bar{2}$			31.0		
22 $\bar{1}$	221, 2 $\bar{2}$ 1	1.989		30.2	47.7	
22 $\bar{3}$	22 $\bar{1}$, 2 $\bar{2}$ 1			17.5		
31 $\bar{2}$	311, 3 $\bar{1}$ 1	1.988	1.992	16.7	33.7	216.8
314	31 $\bar{1}$, 3 $\bar{1}$ 1			17.0		
210	212, 2 $\bar{1}$ 2	1.959	1.963	68.9	135.4	
214	21 $\bar{2}$, 2 $\bar{1}$ 2			66.5		
022	022, 02 $\bar{2}$	1.887	1.881		148.5	166.1
121	122, 1 $\bar{2}$ 2	1.822		2.3	22.2	
123	12 $\bar{2}$, 1 $\bar{2}$ 2			19.9		
030	030	1.813	1.812		14.3	36.5
323	320, 320	1.774		22.2		
13 $\bar{1}$	130, 130	1.756	1.763		31.4	257.5
404	400	1.755			203.9	273.8
031	031, 03 $\bar{1}$	1.714	1.714		0.8	4.1
102	103	1.695	1.697	56.7	69.7	
104	103			13.0		
32 $\bar{2}$	321, 3 $\bar{2}$ 1	1.6802	1.684	54.2	59.5	132.0
324	32 $\bar{1}$, 3 $\bar{2}$ 1			5.3		
414	410, 4 $\bar{1}$ 0	1.6702			2.8	121.5

^a First monoclinic unit cell: $a = 7.02$; $b = 5.44$; $c = 8.76$; $\beta = 143.26^\circ$.

^b Triclinic, pseudo-orthorhombic unit cell: $a = 7.02$; $b = 5.44$; $c = 5.24$.

structure with the Russians reporting that one Na atom has a distorted octahedral and the other an unusual 5-fold coordination. According to our results the structure, although slightly distorted, is truly a tetrahedral structure with every atom having four tetrahedral neighbors at reasonable interatomic distances: Zn-O = 1.95 Å(4×); Si-O = 1.61 Å(4×); Na₁-O = 2.35 Å(4×) and Na₂-O = 2.33 Å(4×).

As stated before, both the parameters obtained by Belov *et al.* and those obtained by us have the same reliability index. In general, one has greater confidence in data obtained from single crystal experiments. Nevertheless, the unusual coordination for Na reported by the Russians as compared to our results and our logical derivation of the Na₂ZnSiO₄ structure from the β-NaFeO₂ structure places perhaps a shadow of a doubt on the correctness of the Russian parameters and makes it highly desirable to have Na₂ZnSiO₄ reinvestigated using single crystal diffraction methods.

Other Isotypic Compounds

No structure refinement has been made on Na₂ZnGeO₄ and Na₂MgGeO₄, however, an intensity calculation using the adjustable parameters for Na₂ZnSiO₄ has shown agreement between observed and calculated intensity values. Due to the different size of the atoms as compared to Na₂ZnSiO₄, there will certainly be small changes of the adjustable parameters, however, the structure type for all three compounds is the same. In analogy to β-NaFeO₂, which undergoes a phase change at 1100C, we found, using thermal analysis, a high temperature transformation with Na₂ZnSiO₄ at 998 ± 4C and Na₂ZnGeO₄ at 1156 ± 4C. The data for Na₂MgGeO₄ are inconclusive. If there is a phase transformation it will be very close to the melting point of Na₂MgGeO₄ which we determined to be at 1178 ± 4C.

Discussion

The structure of Na₂ZnSiO₄ is an example of an ordered quaternary tetrahedral structure based on the hexagonal diamond structure. From the point

of view of the systematization of crystal structures the present study illustrates the usefulness of the concepts of valence electron concentration (Parthé, 1964) and of cross substitution and shows interesting filiations in oxides: C diamond (hexagonal)—ZnO (hexagonal)—γLiAlO₂ (tetragonal) (Bertaut, Delapalme, Bassi, Durif-Varambon, and Joubert, 1965)—βFeNaO₂ (orthorhombic)—Na₂ZnSiO₄ (monoclinic) to take place with a continuous lowering of symmetry.

Theoretically we can predict that only those ordered atom arrangements should occur with tetrahedral structures which require a minimum of electron shift for the necessary *sp*³ hybridization of all the atoms. This leads, in the case of quaternary compounds of composition A₂BCD₄, to two simple atom arrangements; one is the arrangement given here for monoclinic Na₂ZnSiO₄, the other one was recently found with Cu₂CdGeS₄ which has the same relative cell dimensions as the pseudo-orthorhombic Na₂ZnSiO₄ but in effect truly orthorhombic symmetry (Parthé, Yvon, and Deitch, 1969). The limited experimental evidence seems to indicate that the monoclinic structure type is preferred with compounds containing oxygen anions, while the orthorhombic structure type is found with S and Se compounds.

References

1. E. F. BERTAUT AND P. BLUM, *C.R. Acad. Sci. Paris* **239**, 429 (1954).
2. E. F. BERTAUT, A. DELAPALME, AND G. BASSI, *J. Physique* **25**, 545 (1964).
3. E. F. BERTAUT, A. DELAPALME, G. BASSI, A. DURIF-VARAMBON, AND J. C. JOUBERT, *Bull. Soc. Fr. Min. Crist.* **88**, 103 (1965).
4. F. P. BUNDY AND J. S. KASPER, *J. Chem. Phys.* **46**, 3437 (1967).
5. V. V. ILYUKHIN, A. V. NIKITIN, AND N. V. BELOV, *Soviet Physics. Doklady* **11**, 1035 (1967).
6. E. PARTHÉ (1964). "Crystal Chemistry of Tetrahedral Structures." New York: Gordon & Breach. French edition in print with Dunod, Paris.
7. E. PARTHÉ, K. YVON, AND R. H. DEITCH, *Acta Cryst.* **B25**, (1969).